FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

**Department of Mathematics** Dr. Alexander Fromm
**Stochastic Analysis (SS 2019)**

## Exercise sheet 8

**Submission:** 04.06.2019

**Problem*** **(4 additional Points)**

Let $(B_t)_{t\geq 0}$ be a Brownian motion. We define $\tau_1 := \inf\{t \geq 0 \,|\, B_t = 1\}$ and $X_t := \mathbb{1}_{[0,\tau_1]}(t)$, $t \geq 0$.

(a) Show that $X \in \mathcal{L}^2_{loc}(B)$, but $X \notin \mathcal{L}^2(B)$.

(b) Show that $I_\infty(X) = \lim_{t\to\infty} I_t(X)$ is well defined as an a.s.-limit and calculate $\mathbb{E}[I^2_\infty(X)]$. Show that the values $\mathbb{E}[I^2_\infty(X)]$ and $\mathbb{E}\left[\int_0^\infty X_s^2\, ds\right]$ are different.

**Total: 4 additional Points**

**Terms of submission:**

- Solutions can be submitted in groups of at most 2 students.
- Please submit at the beginning of the lecture or until 9:50 a.m. in room 3523, Ernst-Abbe-Platz 2.